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New $\mathcal{N} = 2$ supersymmetric membrane flow in eleven-dimensional supergravity

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Abstract: We construct the 11-dimensional lift of the known $\mathcal{N} = 2$ supersymmetric RG flow solution in 4-dimensional $\mathcal{N} = 8$ gauged supergravity. The squashed and stretched 7-dimensional internal metric preserving $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$ symmetry contains an Einstein-Kahler 2-fold which is a base manifold of 5-dimensional Sasaki-Einstein $Y^{p,q}$ space found in 2004. The nontrivial r(transverse to the domain wall)-dependence of the AdS_4 supergravity fields makes the Einstein-Maxwell equations consistent not only at the critical points but also along the supersymmetric whole RG flow connecting two critical points. With an appropriate 3-form gauge field, we find an exact solution to the 11-dimensional Einstein-Maxwell equations corresponding to the above lift of the $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$-invariant RG flow. The particular limits of this solution give rise to the previous solutions with $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ or $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$.

Keywords: AdS-CFT Correspondence, M-Theory

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1 Introduction

The $\mathcal{N} = 6$ U(N) $\times$ U(N) Chern-Simons matter theory with level k in 3-dimensions is described as the low energy limit of N M2-branes at $\mathbf{C}^4/\mathbf{Z}_k$ singularity [1]. When $k = 1, 2$, the full $\mathcal{N} = 8$ supersymmetry is preserved while for $k > 2$, the supersymmetry is broken to the $\mathcal{N} = 6$ supersymmetry. The matter contents and the superpotential of this theory are the same as for the D3-branes on the conifold [2]. The RG flow between the UV point and the IR point of the 3-dimensional gauge theory can be determined from the gauged $\mathcal{N} = 8$ supergravity in 4-dimensions via AdS/CFT correspondence [3]. The holographic supersymmetric RG flow equation connecting $\mathcal{N} = 8$ SO(8) point to $\mathcal{N} = 2$ SU(3) $\times$ U(1) point has been studied in [4, 5] where the U(1) symmetry here can be identified with U(1)$_R$ symmetry of 3-dimensional theory coming from the $\mathcal{N} = 2$ supersymmetry while those from $\mathcal{N} = 8$ SO(8) point to $\mathcal{N} = 1$ G_2 point has been studied in [5, 6]. The 11-dimensional M-theory lifts of these RG flow equations have been found in [6, 7] by solving the Einstein-Maxwell equations in 11-dimensions with nonzero field strengths in the internal space.

The mass deformed U(2) $\times$ U(2) Chern-Simons matter theory with level $k = 1, 2$ preserving global SU(3) $\times$ U(1)$_R$ symmetry has been studied in [8, 9] while the mass deformation for this theory preserving G_2 symmetry has been described and the nonsupersymmetric RG flow equations preserving SO(7)$^\pm$ symmetries have been discussed in [10]. The holographic RG flow equations connecting $\mathcal{N} = 1$ G_2 point to $\mathcal{N} = 2$ SU(3) $\times$ U(1)$_R$ point have been found in [11]. Moreover, the $\mathcal{N} = 4$ and $\mathcal{N} = 8$ RG flows have been studied and further developments on the gauged $\mathcal{N} = 8$ supergravity in four-dimensions have been done in [12]. Recently, the spin-2 Kaluza-Klein modes around a warped product of AdS_4 and a seven-ellipsoid which has global G_2 symmetry are discussed in [13]. Furthermore, the gauge dual with the symmetry of SU(2) $\times$ SU(2) $\times$ U(1)$_R$ for the second 11-dimensional lift of $\mathcal{N} = 2$ SU(3) $\times$ U(1)$_R$-invariant solution in 4-dimensional supergravity is described in [14].

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The seven-sphere $\mathbf{S}^7$ in the internal space can be realized by $\mathbf{S}^1$-fibration over $\mathbf{CP}^3$ [15] space where the standard Fubini-Study metric on the $\mathbf{CP}^3$ space has $\mathbf{CP}^2$ space [7, 16] or $\mathbf{CP}^1 \times \mathbf{CP}^1$ space [7, 17]. In particular, the U(1) bundle over $\mathbf{CP}^1 \times \mathbf{CP}^1$ space is known as 5-dimensional Sasaki-Einstein $T^{1,1}$ space [18]. In [7], they have found two different 11-dimensional solutions where the first has $\mathbf{CP}^2$ space with $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ symmetry and the second has $\mathbf{CP}^1 \times \mathbf{CP}^1$ space with $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ symmetry. Note that the Ricci tensor for the first solution with frame basis is exactly the same as the one of the second solution, by assuming that the supergravity fields satisfy the same equations of motion discovered by [4]. In other words, the same flow equations in 4-dimensions provide two different 11-dimensional solutions to the equations of the 11-dimensional supergravity.

When we go to 11-dimensional theory from the 4-dimensional gauged supergravity, the various 11-dimensional solutions will occur even if the 4-dimensional flow equations are the same. We expect that since the flow equations in 4-dimensions are related to the $\mathcal{N} = 2$ supersymmetry via $\mathrm{U}(1)_R$ symmetry, other types of 11-dimensional solutions with common 4-dimensional flow equations will arise. One possibility, as we mentioned in [14], is characterized by the $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$ symmetry which is smaller than $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$. The symmetry breaking to $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$ can occur from either the $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ symmetry or $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ symmetry. The metric corresponding to $\mathbf{CP}^1 \times \mathbf{CP}^1$ should preserve only one of two $\mathbf{CP}^1$'s symmetries due to the single SU(2) symmetry.

In this paper, we would like to construct a new 11-dimensional solution preserving the above $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$ symmetry. By assuming that the AdS_4 supergravity fields satisfy the supersymmetric RG flow equations, we should find out the correct 7-dimensional internal space possessing this global symmetry. By realizing that the five-dimensional Sasaki-Einstein $T^{1,1}$ space can be generalized to the 5-dimensional Sasaki-Einstein $Y^{p,q}$ space [19] where p and q are positive integers with $0 \leq q \leq p$, it is obvious to consider this space first. When $p = 1$ and $q = 0$, the $Y^{1,0}$ space is nothing but $T^{1,1}$ space and moreover the isometry of $Y^{p,q}$ is identical to the above $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$. The main procedure given in [7] is to start with the round compactification in terms of U(1)-fibration over the Einstein-Kahler 3-fold, to squash this Einstein-Kahler base ellipsoidally, to stretch the U(1) fiber, and to introduce 3-form tensor gauge potential proportional to the volume form on the base. Inside of Einstein-Kahler 3-fold, one had either $\mathbf{CP}^2$ space or $\mathbf{CP}^1 \times \mathbf{CP}^1$ space. Are there any other Einstein-Kahler 2-folds?

Fortunately, in the construction of $Y^{p,q}$ space, it is known that $Y^{p,q}$ space can be written in terms of U(1) bundle over the Einstein-Kahler 2-fold. Therefore, there is a room for this 4-dimensional Einstein-Kahler 2-fold inside of above Einstein-Kahler 3-fold. Then the next step is to find out the correct 4-form field strengths in this background. Before we use the 11-dimensional Einstein-Maxwell equations directly, it is better to imitate the 3-forms appeared in previous $\mathbf{CP}^2$ or $\mathbf{CP}^1 \times \mathbf{CP}^1$ cases. Basically the structure of 3-form from the triple wedge product between the orthonormal frames looks similar to each other. The overall functional dependence on the AdS_4 supergravity fields and the exponential factors corresponding to unbroken U(1) symmetries can be determined by solving the 11-dimensional Einstein-Maxwell equations directly.

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In section 2, starting with the two parts of $Y^{p,q}$ space metric, U(1) bundle and the 4-dimensional base space which is Einstein-Kahler 2-fold, we put them inside of the squashed and stretched 7-dimensional internal space appropriately. Then one determines the full 11-dimensional metric with the correct warp factor. Assuming that the two AdS_4 supergravity fields satisfy the domain wall solutions, one computes the Ricci tensor in this background completely. For the 4-form field strengths, one makes an ansatz by writing the three parts, 1) the overall function, 2) the exponential function with U(1)'s and 3) the triple wedge product between the orthonormal frames. Eventually, the 11-dimensional Einstein-Maxwell equations determine all the undetermined quantities.

In section 3, we summarize the results of this paper and make some future directions.

In the appendix, we present the detailed expressions for the Ricci tensor and 4-form field strengths.

2 An $\mathcal{N}=2$ supersymmetric $\mathrm{SU}(2)\times U(1)\times U(1)_R$-invariant flow in an 11-dimensional theory

When the 11-dimensional supergravity is reduced to 4-dimensional $\mathcal{N}=8$ gauged supergravity, the 4-dimensional spacetime metric contains a warp factor which depends on both 4-dimensional spacetime coordinates and 7-dimensional internal space coordinates. The internal metric of deformed seven-sphere can be obtained from the AdS_4 supergravity data, the supergravity fields (ρ, χ), using the explicit formula [20], and the warp factor is also determined. We have

$$ds_{11}^2 = \Delta(r,\mu)^{-1}\left(dr^2 + e^{2A(r)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu\right) + L^2\sqrt{\Delta(r,\mu)}\,ds_7^2(\rho,\chi), \tag{2.1}$$

where the 3-dimensional metric is given by $\eta_{\mu\nu}=(-,+,+)$, the radial variable $r=x^4$ is the coordinate transverse to the domain wall, the scale factor $A(r)$ behaves linearly in r at UV and IR regions, L is a radius of round seven-sphere $\mathbf{S}^7$ and the warp factor $\Delta(r,\mu)$ also depends on the μ that is one of the internal coordinates($\mu = x^5$) as well as the radial coordinate r via the supergravity fields (ρ,χ).

Let us assume that the supergravity fields (ρ,χ) in 4-dimensions satisfy the supersymmetric RG flow equations [4] with $\mathrm{SU}(3)\times\mathrm{U}(1)_R$ symmetry in the convention of [7]:

$$\begin{aligned}
\frac{d\rho}{dr} &= \frac{1}{8L\,\rho}\left[(\cosh(2\chi)+1)+\rho^8\,(\cosh(2\chi)-3)\right],\\
\frac{d\chi}{dr} &= \frac{1}{2L\,\rho^2}\,(\rho^8-3)\,\sinh(2\chi),\\
\frac{dA}{dr} &= \frac{1}{4L\,\rho^2}\left[3(\cosh(2\chi)+1)-\rho^8(\cosh(2\chi)-3)\right].
\end{aligned} \tag{2.2}$$

In 4-dimensions, there exist two critical points, $\mathcal{N}=8$ SO(8) critical point at which $(\rho,\chi)=(1,0)$ and $\mathcal{N}=2$ $\mathrm{SU}(3)\times\mathrm{U}(1)_R$ critical point at which $(\rho,\chi)=(3^{\frac{1}{8}},\frac{1}{2}\cosh^{-1}2)$. One can easily check that at these two points, $\frac{d\rho}{dr}$ and $\frac{d\chi}{dr}$ vanish due to the right hand sides of (2.2) are equal to zero. Furthermore, the criticality can be observed from the fact that the first two right hand sides of (2.2) can be written as the derivatives of superpotential $W(\rho,\chi)$

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with respect to the field ρ and the field χ respectively. One can read off the superpotential $W(\rho,\chi)$ explicitly by realizing that the right hand side of third equation in (2.2) is equal to $-\frac{2}{L}W(\rho,\chi)$. The superpotential has 1 and $\frac{3^{\frac{3}{4}}}{2}$ at two critical values respectively. We will see the 11-dimensional lift of this superpotential, geometric superpotential, later when we discuss about the 11-dimensional field equations.

We need to find out the correct 7-dimensional metric which preserves $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$ symmetry which maybe obtained from the symmetry breaking of above bigger symmetry $\mathrm{SU}(3)\times\mathrm{U}(1)_R$ corresponding to the stretched five-sphere $\mathbf{S}^5$ described by U(1) bundle over the $\mathbf{CP}^2$ or $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_R$ symmetry corresponding to the stretched $T^{1,1}$ space realized by U(1) bundle over $\mathbf{CP}^1\times\mathbf{CP}^1$. Once we have found this 7-dimensional internal metric with the warp factor given in [7], then the full 11-dimensional metric can be written as (2.1). Then how one can find this internal metric with the above specific symmetry? It is not obvious that the SU(2) symmetry among the full $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$ symmetry is realized from $\mathbf{CP}^2$ space which preserves SU(3) symmetry. However, any $\mathbf{CP}^1$ factor in $\mathbf{CP}^1\times\mathbf{CP}^1$ space can provide this SU(2) symmetry because the $\mathbf{CP}^1$ preserves the SU(2) symmetry. So, our strategy is to look at the second solution of [7] closely rather than the first solution.

At first, let us replace the 4-dimensional $\mathbf{CP}^1\times\mathbf{CP}^1$ space appearing in the 7-dimensional internal space in [7] with the 4-dimensional Einstein-Kahler 2-fold which lives in the five-dimensional $Y^{p,q}$ space [19]. Next, we need to find out the correct one-form which contains the U(1) bundle over this Einstein-Kahler 2-fold. This one-form ω is given by[1]

$$\omega=\frac{1}{2}\sin(2\mu)\left[-\frac{1}{\rho(r)^4}\,d\alpha+\rho(r)^4\,(u,Jdu)\right], \tag{2.3}$$

where we introduce the $\mathbf{R}^8$ vector $u=(u^1,\cdots,u^6,0,0)$ which parametrize a unit $\mathbf{S}^5$ sphere and J is the Kahler form with $J_{12}=J_{34}=J_{56}=J_{78}=1$. The product (u,Jdu) is defined as $(u,Jdu)\equiv u^A J_{AB}u^B$. Note that the one-form in subsection 4.1 of [7] is the U(1) bundle over the 4-dimensional $\mathbf{CP}^2$ space while the one-form in eq. (4.38) of [7] is the U(1) bundle over the 4-dimensional $\mathbf{CP}^1\times\mathbf{CP}^1$ space.

What is (u,Jdu) in (2.3) corresponding to the U(1) bundle over the Einstein-Kahler 2-fold? Let us recall the metric for the 5-dimensional Sasaki-Einstein space $Y^{p,q}$ with $c=1$ [19]

$$\begin{aligned} ds^2_{Y^{p,q}} &= ds^2_{EK(2)}+\frac{1}{9}\left[(d\psi-\cos\theta\,d\phi)+y\,(d\beta+\cos\theta\,d\phi)\right]^2 \\ &= \left[\frac{1}{6}\,(1-y)\,(d\theta^2+\sin^2\theta\,d\phi^2)+\frac{1}{w(y)q(y)}\,dy^2+\frac{1}{36}w(y)q(y)\,(d\beta+\cos\theta\,d\phi)^2\right] \\ &\quad +\frac{1}{9}\left[(d\psi-\cos\theta\,d\phi)+y\,(d\beta+\cos\theta\,d\phi)\right]^2, \end{aligned} \tag{2.4}$$

where y-dependent functions are given by

$$w(y)\equiv\frac{2(a-y^2)}{1-y},\qquad q(y)\equiv\frac{a-3y^2+2y^3}{a-y^2},\qquad a=\frac{1}{2}-\frac{(p^2-3q^2)}{4p^3}\sqrt{4p^2-3q^2}. \tag{2.5}$$

[1] The 11-th coordinate α here corresponds to ψ introduced in [7].

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Also note that the form in the last line of (2.4) provides the Kahler 2-form and satisfies

$$\frac{1}{6}d\left[-\cos\theta\, d\phi + y(d\beta+\cos\theta\, d\phi)\right] = \frac{1}{6}\left(1-y\right)\, \sin\theta\, d\theta\wedge d\phi + \frac{1}{6}\, dy\wedge(d\beta+\cos\theta\, d\phi). \tag{2.6}$$

Then it is natural to view that we identify (u, Jdu) with the U(1) bundle over this Einstein-Kahler 2-fold as follows:

$$(u, Jdu) = \frac{1}{3}\left[(d\psi - \cos\theta\, d\phi) + y\,(d\beta+\cos\theta\, d\phi)\right]. \tag{2.7}$$

By plugging (2.7) into (2.3), we have one-form ω explicitly.

Finally, we should write down the U(1) Hopf fiber (x, Jdx) on $\mathbf{CP}^3$(note that for $\rho=1$ and $\chi=0$, the internal metric should contain a $\mathbf{CP}^3$ factor) where $x=(x^1,\cdots,x^8)$ is a vector on $\mathbf{R}^8$ in terms of (u, Jdu) using [7]

$$(x, Jdx) = \cos^2\mu\,(u, Jdu) + \sin^2\mu\, d\alpha. \tag{2.8}$$

One also introduces another vector $v=(0,\cdots,0,\cos\alpha,\sin\alpha)$ in $\mathbf{R}^8$ and then the above x can be written as $x = u\cos\mu + v\sin\mu$. In (2.8), we used $(v, Jdv) = d\alpha$. The 7-dimensional internal space metric $ds^2(\rho,\chi)$ without a warp factor can be written as

$$ds_7^2(\rho,\chi) = \rho(r)^{-4}\xi^2 d\mu^2 + \rho(r)^2\cos^2\mu\, ds^2_{EK(2)} + \xi^{-2}\omega^2 + \xi^{-2}\cosh^2\chi(r)(x, Jdx)^2. \tag{2.9}$$

Here we substituted the metric (2.4) for the Einstein-Kahler 2-fold where the four coordinates are parametrized by $(\theta = x^6, \phi = x^7, y = x^8, \beta = x^9)$ in the second term of (2.9). After plugging the 1-form (2.3) with (2.7) in the third term of (2.9) and the U(1) Hopf fiber (2.8) in the last term of (2.9), we obtain the final 7-dimensional internal metric preserving $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$ symmetry as follows:

$$\begin{aligned} ds_7^2(\rho,\chi) &= \rho(r)^{-4}\xi(r,\mu)^2 d\mu^2 \\ &+\rho(r)^2\cos^2\mu\left[\frac{1}{6}(1-y)(d\theta^2+\sin^2\theta d\phi^2)+\frac{1}{w(y)q(y)}dy^2+\frac{1}{36}w(y)q(y)(d\beta+\cos\theta d\phi)^2\right] \\ &+\xi(r,\mu)^{-2}\frac{1}{4}\sin^2(2\mu)\left[-\frac{1}{\rho(r)^4}d\alpha+\rho(r)^4\frac{1}{3}\left[(d\psi-\cos\theta d\phi)+y(d\beta+\cos\theta d\phi)\right]\right]^2 \\ &+\xi(r,\mu)^{-2}\cosh^2\chi(r)\left[\sin^2\mu d\alpha+\cos^2\mu\frac{1}{3}\left[(d\psi-\cos\theta d\phi)+y(d\beta+\cos\theta d\phi)\right]\right]^2, \end{aligned} \tag{2.10}$$

where the quadratic form $\xi^2 \equiv (x, Qx)$ with $Q = \mathrm{diag}(\rho(r)^{-2},\cdots,\rho(r)^{-2},\rho(r)^6,\rho(r)^6)$ in 8-dimensional space can be computed and it is given by [7]

$$\xi(r,\mu) = \frac{\sqrt{X(r,\mu)}}{\rho(r)}, \qquad X(r,\mu)\equiv\cos^2\mu+\rho(r)^8\,\sin^2\mu.$$

In (2.10), we explicitly presented the r-dependence in every place. The nontrivial squashing characterized by $\rho(r)$ deforms the metric on the $\mathbf{CP}^3$(by changing the variables appropriately [19] one makes the 5-dimensional metric on $Y^{p,q}$ space as a U(1) bundle over the

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Fubini-Study metric on $\mathbf{CP}^2$, one obtains the usual round 5-sphere $\mathbf{S}^5$ and the first three lines of (2.10) contain $\mathbf{CP}^3$ metric) and moreover rescales the Hopf fiber which appears in the last line of (2.10). The stretching is characterized by $\chi(r)$. However, there exists $SU(2)\times U(1)$ symmetry from the structure of Einstein-Kahler 2-fold in $ds^2_{EK(2)}$. The U(1) symmetry is generated by the angle β. The combined two U(1) symmetries by the angle $\psi(=x^{10})$ and the angle $\alpha(=x^{11})$ will provide a single $U(1)_R$ symmetry which is relevant to the $\mathcal{N}=2$ supersymmetry. We will return to this issue when we discuss about the 4-form field strengths later.

For $\mu=0$, the 7-dimensional metric (2.10) reduces to the following metric on moduli space for the M2-brane probe

$$\rho(r)^2 ds^2_{Y^{p,q}} + \rho(r)^2 \sinh^2\chi(r)\,\frac{1}{9}\left[(d\psi-\cos\theta\,d\phi)+y(d\beta+\cos\theta\,d\phi)\right]^2, \tag{2.11}$$

where the metric for $Y^{p,q}$ is given by (2.4). In particular, the $\mathbf{S}^5$ or $T^{1,1}$ is replaced by $Y^{p,q}$ and for large r the moduli space (2.11) approaches the Ricci-flat conifold. Now one sees that the function $\sinh^2\chi(r)$ plays the role of a stretching of the U(1)-fiber. Then on can say, for this particular coordinate $\mu=0$, there exists a stretched five-sphere $\mathbf{S}^5$, a stretched $T^{1,1}$ space or a stretched $Y^{p,q}$ space depending on the U(1)-fibers.

From these observations so far, we obtain the following set of frames for the 11-dimensional metric (2.1):

$$\begin{aligned}
e^1 &= -\frac{1}{\sqrt{\Delta(r,\mu)}}\,e^{A(r)}\,dx^1, \qquad e^2 = \frac{1}{\sqrt{\Delta(r,\mu)}}\,e^{A(r)}\,dx^2, \qquad e^3 = \frac{1}{\sqrt{\Delta(r,\mu)}}\,e^{A(r)}\,dx^3,\\
e^4 &= \frac{1}{\sqrt{\Delta(r,\mu)}}\,dr,\\
e^5 &= L^2\,\sqrt[4]{\Delta(r,\mu)}\,\frac{\sqrt{X(r,\mu)}}{\rho(r)^3}\,d\mu,\\
e^6 &= L^2\,\sqrt[4]{\Delta(r,\mu)}\,\rho(r)\cos\mu\,\sqrt{\frac{1-y}{6}}\,d\theta,\\
e^7 &= L^2\,\sqrt[4]{\Delta(r,\mu)}\,\rho(r)\cos\mu\,\sqrt{\frac{1-y}{6}}\,\sin\theta\,d\phi,\\
e^8 &= L^2\,\sqrt[4]{\Delta(r,\mu)}\,\rho(r)\cos\mu\,\frac{1}{\sqrt{w(y)\,q(y)}}\,dy,\\
e^9 &= L^2\,\sqrt[4]{\Delta(r,\mu)}\,\rho(r)\cos\mu\,\frac{1}{6}\,\sqrt{w(y)\,q(y)}\,(d\beta+\cos\theta\,d\phi),\\
e^{10} &= L^2\,\sqrt[4]{\Delta(r,\mu)}\frac{\rho(r)}{\sqrt{X(r,\mu)}}\frac{1}{2}\sin(2\mu)\left[-\frac{d\alpha}{\rho(r)^4}+\frac{1}{3}\rho(r)^4\left[(d\psi-\cos\theta d\phi)+y(d\beta+\cos\theta d\phi)\right]\right],\\
e^{11} &= L^2\,\sqrt[4]{\Delta(r,\mu)}\frac{\rho(r)\cosh\chi(r)}{\sqrt{X(r,\mu)}}\left[\sin^2\mu d\alpha+\frac{1}{3}\cos^2\mu\left[(d\psi-\cos\theta d\phi)+y(d\beta+\cos\theta d\phi)\right]\right],
\end{aligned} \tag{2.12}$$

where the warp factor is given by [7]

$$\Delta(r,\mu) = \frac{\rho(r)^{\frac{4}{3}}}{X(r,\mu)^{\frac{2}{3}}\,\cosh^{\frac{4}{3}}\chi(r)}. \tag{2.13}$$

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The constant L in (2.12) in the 7-dimensional internal space is determined by using the symmetry of UV fixed point later.

Denoting the 11-dimensional metric as g_{MN} with the convention $(-,+,\cdots,+)$ and the antisymmetric tensor fields as F_{MNPQ}, the Einstein-Maxwell equations are given by [21]

$$
\begin{aligned}
R_M{}^N &= \frac{1}{3} F_{MPQR}F^{NPQR} - \frac{1}{36}\delta_M^N F_{PQRS}F^{PQRS}, \\
\nabla_M F^{MNPQ} &= -\frac{1}{576} E\,\epsilon^{NPQRSTUVWXY} F_{RSTU}F_{VWXY},
\end{aligned}
\tag{2.14}
$$

where the covariant derivative ∇_M on F^{MNPQ} in (2.14) is given by $E^{-1}\partial_M(EF^{MNPQ})$ together with elfbein determinant $E \equiv \sqrt{-g_{11}}$. The epsilon tensor $\epsilon_{NPQRSTUVWXY}$ with lower indices is purely numerical. All the indices are based on the coordinate basis.

At the SO(8)-invariant UV fixed point [4]. For

$$
\rho(r) = 1, \qquad \chi(r) = 0, \tag{2.15}
$$

one should recover the maximally symmetric $AdS_4 \times \mathbf{S}^7$ solution. In general, one can introduce the arbitrary coefficients in the frames e^6 to e^{11} of (2.12). But these can be fixed in order to make the Ricci tensor have the form

$$
R_M^N = \frac{6}{L^2}\,\mathrm{diag}(-2,-2,-2,-2,1,1,1,1,1,1,1),
$$

which fixes the round $\mathbf{S}^7$ radius to be L, twice the AdS_4 radius, as expected. As Freund-Rubin parametrization [22], the 3-form gauge field with 3-dimensional M2-brane indices maybe defined by [7]

$$
A^{(3)} = \frac{1}{2} e^{\frac{6r}{L}}\, dx^1 \wedge dx^2 \wedge dx^3. \tag{2.16}
$$

Note that at the UV end of the flow the function $A(r)$ behaves as $\frac{2}{L}r$ from the solution (2.2) for $A(r)$ and $W = 1$. The exponential factor $e^{3A(r)}$ will be compensated by the same factor from the 11-dimensional metric when we derive the geometric superpotential along the flow. From (2.16), one obtains the only nonzero component for the 4-form as $F_{1234} = -\frac{18}{L}$.

At the $\mathbf{SU(2)\times U(1)\times U(1)}_{\boldsymbol{R}}$-invariant IR fixed point [4] As we mentioned before, there exists IR critical point characterized by

$$
\rho(r) = 3^{\frac{1}{8}}, \qquad \chi(r) = \frac{1}{2}\cosh^{-1} 2. \tag{2.17}
$$

The 3-form gauge field with 3-dimensional M2-brane indices can be constructed as the UV critical point. By realizing that at the IR end of the flow the function $A(r)$ behaves as $\frac{3^{\frac{3}{4}}}{L}r$ from the solution (2.2) for $A(r)$ and $W = \frac{3^{\frac{3}{4}}}{2}$(and we define $\hat{L} \equiv 3^{-\frac{3}{4}}L$), one writes down the 3-form gauge field including the internal parts as follows [7]:

$$
A^{(3)} = \frac{3^{\frac{3}{4}}}{4} e^{\frac{3r}{\hat{L}}}\, dx^1 \wedge dx^2 \wedge dx^3 + C^{(3)} + (C^{(3)})^*. \tag{2.18}
$$

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How does one determine the internal 3-form field $C^{(3)}$? The Kahler form in (2.6) contains $e^6 \wedge e^7$ and $e^8 \wedge e^9$ that lead to the natural basis of the one-forms and the $\mathbf{CP}^3$ factor for $\rho = 1$ and $\chi = 0$ has also e^5 and e^{10} which can be combined together. In fact, we find

$$C^{(3)} = -\frac{1}{4} \sinh \chi(r)\, e^{i(\alpha+\psi)}\, (e^5 - ie^{10}) \wedge (e^6 + ie^7) \wedge (e^8 + ie^9). \tag{2.19}$$

In general, the overall function depends on both $\rho(r)$ and $\chi(r)$. However, the above expression (2.19) possesses only $\chi(r)$-dependence. The coefficients for α and ψ in the exponent are fixed as 1 and 1 respectively. We have considered the angle β also in the exponent but the coefficient for this vanishes from 11-dimensional Einstein equation. Although the structure of triple product (2.19) between the orthonormal basis looks very similar to the previous constructions with $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ symmetry or $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ symmetry(up to signs), the functional behavior of the exponential function, i.e., the rotations with $\mathrm{U}(1)$ symmetries in the fields behave differently. It is interesting note that the overall function contains $\sinh \chi(r)$ which plays the role of a stretching $\mathrm{U}(1)$ fiber we described before.

Let us explain all these in detail. Let us go to the Ricci tensor first in the frame basis we introduced in (2.12). The Ricci tensor has only two nonvanishing off-diagonal components:R_{10}^{11} and R_{11}^{10}. There exists a nontrivial identity between these components. It turns out the Ricci tensor is identical to the one with $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ symmetry or the one with $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ symmetry. That is, the Ricci tensor for three cases has same value (in the frame basis) at the IR critical point. Let us present them here for convenience:

$$\begin{aligned}
R_1^1 &= -\frac{(55 - 32\cos 2\mu + 3\cos 4\mu)}{3 \cdot 2^{\frac{1}{3}} \sqrt{3} \hat{L}^2 (2 - \cos 2\mu)^{\frac{8}{3}}} = R_2^2 = R_3^3 = R_4^4 = -2R_6^6 = -2R_7^7 = -2R_8^8 = -2R_9^9, \\
R_5^5 &= \frac{(29 - 16\cos 2\mu)}{3 \cdot 2^{\frac{1}{3}} \sqrt{3} \hat{L}^2 (2 - \cos 2\mu)^{\frac{8}{3}}} = R_{10}^{10}, \qquad R_{10}^{11} = -\frac{2 \cdot 2^{\frac{1}{6}} \sin 2\mu}{\sqrt{3} \hat{L}^2 (2 - \cos 2\mu)^{\frac{5}{3}}} = R_{11}^{10}, \\
R_{11}^{11} &= \frac{(80 - 64\cos 2\mu + 9\cos 4\mu)}{3 \cdot 2^{\frac{1}{3}} \sqrt{3} \hat{L}^2 (2 - \cos 2\mu)^{\frac{8}{3}}}.
\end{aligned} \tag{2.20}$$

All these depend on only $\mu (= x^5)$ coordinate. One can also obtain the Ricci tensor in coordinate basis that depends on $y (= x^8)$ and $\theta (= x^6)$ as well as μ. Now it is ready to use the 11-dimensional Einstein equation which is the first one of (2.14) where the indices are based on the coordinate basis. One can transform this Einstein equation with coordinate basis into the one with frame basis without any difficulty via (2.12). The $(10, 9)$ component of right hand side of Einstein equation is nonzero in general but the corresponding R_{10}^9 from (2.20), which appears in the left hand side of Einstein equation, vanishes. This implies that the coefficient of β should vanish and the coefficient of ψ should be 1 in the exponent of 3-form (2.19). Then there exists a $\mathrm{U}(1)$ symmetry generated by the angle β. Furthermore, by comparing the $(10, 11)$ component of Einstein equation, the coefficient for the angle α which is equal to 1 and the overall coefficient of 3-form that is $-\frac{1}{4}$ are completely fixed. At the moment, one cannot determine the functional dependence for $\sinh \chi(r)$ in (2.19) because we are looking for the behavior at the critical point (2.17). We return to this issue when we discuss about the RG flow later.

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The internal part of $F^{(4)}$ can be written as $dC^{(3)} + d(C^{(3)})^*$. The antisymmetric tensor fields can be obtained from $F^{(4)} = dA^{(3)}$ with (2.18). It turns out that the antisymmetric field strengths have the following nonzero components in the orthonormal frame basis used in (2.19) or in (2.12)

$$\begin{aligned}
F_{1234} &= -\frac{3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{3}{4}}}{\hat{L}(2-\cos 2\mu)^{\frac{4}{3}}}, \qquad F_{568\,10} = \frac{2^{\frac{1}{3}} \cdot 3^{\frac{3}{4}} \sin(\alpha+\psi) \sin 2\mu}{\hat{L}(2-\cos 2\mu)^{\frac{4}{3}}}, = -F_{579\,10}, \\
F_{568\,11} &= -\frac{2^{\frac{5}{6}} \cdot 3^{\frac{3}{4}} \sin(\alpha+\psi)}{\hat{L}(2-\cos 2\mu)^{\frac{1}{3}}} = -F_{579\,11} = F_{69\,10\,11} = F_{78\,10\,11}, \\
F_{569\,10} &= \frac{2^{\frac{1}{3}} \cdot 3^{\frac{3}{4}} \cos(\alpha+\psi) \sin 2\mu}{\hat{L}(2-\cos 2\mu)^{\frac{4}{3}}} = F_{578\,10}, \\
F_{569\,11} &= -\frac{2^{\frac{5}{6}} \cdot 3^{\frac{3}{4}} \cos(\alpha+\psi)}{\hat{L}(2-\cos 2\mu)^{\frac{1}{3}}} = F_{578\,11} = -F_{68\,10\,11} = F_{79\,10\,11},
\end{aligned} \tag{2.21}$$

where the angle-dependences for α and ψ appear in the combination of $(\alpha+\psi)$ as observed previously. One can make the two U(1) symmetries generated by α and ψ which preserve this combination $(\alpha+\psi)$. Note that these 4-forms break the $\mathrm{SU}(2) \times \mathrm{U}(1)_\beta \times \mathrm{U}(1)_\alpha \times \mathrm{U}(1)_\psi$ into $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$ where U(1) is generated by the angle β. It is obvious that the invariance of $\mathrm{U}(1)_\beta$ comes from the fact that (2.21) do not depend on the angle β as we explained before. After substituting (2.21) into the right hand side of Einstein equation (2.14) with frame basis (2.12) one reproduces the one of $\mathrm{SU}(3) \times \mathrm{U}(1)_R$ case [7] or $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ exactly. This feature is also expected since as we already mentioned, the Ricci tensor for three independent cases is identical to each other. In other words, the 4-forms themselves are different from each other, their combinations appearing in the right hand side of Einstein equation are the same. In particular, the 4-form given in (2.21) looks very similar to the one of $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ symmetry case: same independent components up to signs.

Along the $\mathbf{SU(2) \times U(1) \times U(1)_R}$-invariant RG flow. The nontrivial r-dependence of supergravity fields (ρ, χ) via (2.2) requires that the 11-dimensional Einstein-Maxwell equations become consistent with not only at the critical points but also along the supersymmetric RG flow connecting the two critical points. For solutions with varying scalars, the ansatz for the 4-form field strength will be more complicated. We will apply the correct ansatz for the 11-dimensional 3-form gauge field by acquiring the r-dependence of the supergravity scalars and will derive the 11-dimensional Einstein-Maxwell equations corresponding to the $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$-invariant RG flow.

Let us take the 3-form ansatz as follows [7]:

$$A^{(3)} = \widetilde{W}(r,\mu)\, e^{3A(r)}\, dx^1 \wedge dx^2 \wedge dx^3 + C^{(3)} + (C^{(3)})^*, \tag{2.22}$$

where $C^{(3)}$ is given by (2.19) as before. Then how does one determine the $\chi(r)$ dependence appearing this 3-form? One puts an arbitrary function $f(\rho(r), \chi(r))$ in front of this 3-form at the beginning. One also obtains the Ricci tensor from the 11-dimensional metric (2.1) when the supergravity fields $(\rho(r), \chi(r))$ vary with respect to the r-coordinate.

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They are given in (A.1) of the appendix A where all the derivative terms before using the flow equations disappear by constraining the conditions (2.2). When one needs to have the second derivative terms for $\rho(r), \chi(r)$ or $A(r)$, one should differentiate the flow equations further and change the right hand side by using the flow equations again and removing the derivative terms. The $(10, 11)$ component of Einstein equation determines the function $f(\rho(r), \chi(r))$. The R_{10}^{11} component is given in (A.1) while the corresponding right hand side depends on this function and its derivative. One obtains $v(r) f(v(r)) + (1 - v(r)^2) f'(v(r)) = 0$ where $v(r) \equiv \cosh\chi(r)$. This implies that the solution $f(v(r))$ is exactly the same as $\sinh\chi(r)$.

Now let us determine the exact form for the geometric superpotential introduced in (2.22). Let us consider $(4,4), (4,5)$ and $(5,5)$ components of Einstein equation. The first and last ones contain $\widetilde{W}^2, \widetilde{W}\partial_r\widetilde{W}, (\partial_r\widetilde{W})^2$ and $(\partial_\mu\widetilde{W})^2$ while the second one contains $\widetilde{W}\partial_\mu\widetilde{W}$ and $\partial_r\widetilde{W}\partial_\mu\widetilde{W}$. By eliminating $(\partial_r\widetilde{W})^2$ from $(4,4)$ and $(5,5)$ components, one obtains

$$\frac{\widetilde{W}(r,\mu)}{\partial\mu} = -\frac{1}{2\rho(r)^2}\left[\cosh^2\chi(r) + \rho(r)^8(-2 + \cosh^2\chi(r))\right]\sin 2\mu. \tag{2.23}$$

By integrating this (2.23) with respect to the μ coordinate, one gets

$$\widetilde{W}(r,\mu) = \frac{1}{4\rho(r)^2}\left[\cosh^2\chi(r) + \rho(r)^8(-2 + \cosh^2\chi(r))\right]\cos 2\mu + g(r), \tag{2.24}$$

where $g(r)$ is an arbitrary function of r. How one can determine the function $g(r)$? By recalling that the superpotential $W(\rho,\chi)$ in 4-dimensions has terms like $\rho(r)^6$ or $\rho(r)^{-2}$. Then one makes further ansatz for $g(r)$ as $g(r) = \rho(r)^{-2}\, h_1(\chi(r)) + \rho(r)^6\, h_2(\chi(r))$. Let us insert these into the $(4,5)$ component of Einstein equation. Then the unknown functions $h_1(\chi(r))$ and $h_2(\chi(r))$ are completely fixed and they are given by

$$h_1(\chi(r)) = \frac{1}{4}\cosh^2\chi(r), \qquad h_2(\chi(r)) = \frac{1}{8}\left(3 - \cosh 2\chi(r)\right). \tag{2.25}$$

By plugging these (2.25) into (2.24) with $g(r)$ above, one obtains the final expression for the geometric superpotential as follows:

$$\widetilde{W}(r,\mu) = \frac{1}{4\rho(r)^2}\left[(\cosh 2\chi(r) + 1)\cos^2\mu - \rho(r)^8\left(\cosh 2\chi(r) - 3\right)\sin^2\mu\right], \tag{2.26}$$

which is exactly the same as the one [7] found in other two cases. When $\cos^2\mu = \frac{3}{4}$, then we have $\widetilde{W}(r,\mu) = -\frac{1}{2}W(\rho,\chi)$ where $W(\rho,\chi)$ is a superpotential in 4-dimensions.

Comparing with the previous 4-form fields at the IR fixed point, the mixed 4-form fields $F_{\mu\nu\rho 5}, F_{4mnp}$ and F_{45mn} where $\mu,\nu,\rho = 1,2,3$ and $m,n,p = 6,7,\cdots,11$ are new if we look at the (B.1). Indeed, they are not forbidden to occur by the global symmetry once we suppose that the 4-dimensional metric has the domain wall factor $e^{3A(r)}$ that breaks the 4-dimensional conformal invariance. At both UV and IR critical points, the 4-dimensional spacetime becomes asymptotically AdS_4 which has conformal invariance and the mixed field strengths should vanish there.

In order to check the remaining Maxwell equation, one needs to know the elfbein determinant $E = \sqrt{-g_{11}}$ and it is given by

$$E = \frac{1}{4\,\rho(r)^{\frac{4}{3}}} 9 \cdot 3^{\frac{1}{4}}\, e^{3A(r)}\, \hat{L}^7\, \cosh^{\frac{4}{3}}\chi(r)\,(y-1)\,\cos^5\mu\ \sin\theta\,\sin\mu\,\left(\cos^2\mu + \rho(r)^8\,\sin^2\mu\right)^{\frac{2}{3}},$$

by computing the determinant of 11-dimensional metric (2.1). The right hand side of Maxwell equation of (2.14) contains also the determinant of 11-dimensional inverse metric. Written in terms of coordinate basis, one should also transform the 4-forms in (B.1) with frame basis of the appendix B into the ones with coordinate basis via e^a_m appearing in (2.12). On the other hand, the left hand side of Maxwell equation has 4-form with upper indices which can be determined by using the 11-dimensional inverse metric and 4-forms with lower indices in the coordinate basis. We do not present them here because they are rather complicated. We have checked that all of the Maxwell equations of motion are indeed satisfied.

Thus we have established that the solutions (2.22), (2.19), and (2.26) actually consists of an exact solution to the 11-dimensional supergravity characterized by bosonic field equations (2.14), provided that the deformation parameters $(\rho(r), \chi(r))$ of the 7-dimensional internal space and the domain wall amplitude $A(r)$ develop in the AdS_4 radial direction along the $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$-invariant RG flow (2.2).

So far, we have focused on $c = 1$(in other words, for $c \neq 0$ one can rescale y to set $c = 1$ and the metric has one parameter family characterized by a) in the $\mathbf{S}^2$ metric of (2.4) and the coefficient is given by $(1-y)$. For $c = 0$ where a is a trivial rescaling parameter, then the metric of (2.4) leads to the standard metric of $T^{1,1}$ space. Then one can follow the procedure for the second solution in [7]. On the other hand, for $a = 1$ where a is defined in (2.5) and c is a trivial rescaling parameter, the metric provides the round five-sphere $\mathbf{S}^5$ metric. Then one takes the first solution of [7]. Schematically, we draw these solutions in figure 1. In 11-dimensional view point, the three independent RG flows characterized by

$$\begin{aligned} \mathbf{S}^5 - \text{flow} &: \mathrm{SU}(3)\times\mathrm{U}(1)_R, \\ T^{1,1} - \text{flow} &: \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_R, \\ Y^{p,q} - \text{flow} &: \mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R, \end{aligned}$$

arrive at the IR fixed point at which they have common Ricci tensor (2.20). Depending on their global symmetry, the internal 3-forms, in each case, have the right structures in the exponential function with common $\sinh\chi(r)$ dependence. However, the 3-form in the M2-brane world-volume directions with the same geometric superpotential (2.26) is common to three different solutions. It is surprising that although the 4-forms are different from each other completely, the squares of these 4-forms appearing in the right hand side of Einstein equation (2.14) give rise to the same expressions.

3 Conclusions and outlook

We have derived the 11-dimensional Einstein-Maxwell equations corresponding to the $\mathcal{N} = 2$ $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$-invariant RG flow in the 4-dimensional gauged supergravity. The AdS_4 supergravity fields can be interpreted as the geometric parameters for the

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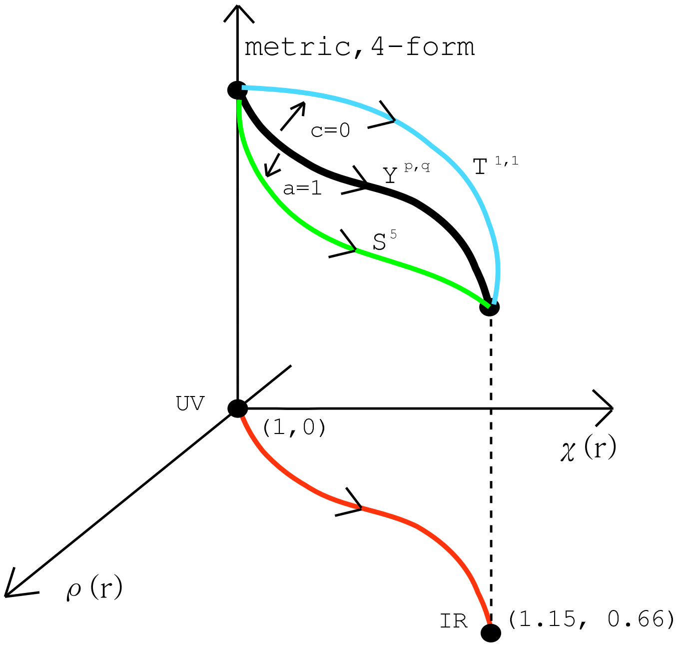

Figure 1. The RG flow starting from SO(8) UV fixed point to $\mathrm{SU}(3)\times\mathrm{U}(1)_R$ IR fixed point and its three 11-dimensional lifts. The theory(in lower plane) flows a $\mathrm{SU}(3)\times\mathrm{U}(1)_R$-invariant fixed point at which $(\rho,\chi) = (1.15, 0.66)$ as they vary with respect to r according to (2.2) [4] starting from $(\rho,\chi) = (1,0)$. In its 11-dimensional lift, there exist three flows. At each curve, the 11-dimensional metric and 4-forms vary with the deformation parameters (ρ,χ). The solutions to the upper and lower ones were found in [7] while the solution to the middle one is found in this paper. The Ricci tensor for three curves is the same and given in the appendix A. The common $\mathrm{U}(1)_R$ factor in the global symmetry plays the role of $\mathcal{N} = 2$ supersymmetry along the whole three flows. Either $\mathbf{S}^5$-flow or $T^{1,1}$-flow can be obtained from the more general $Y^{p,q}$-flow by taking the limit $a = 1$ or $c = 0$ respectively.

7-dimensional internal space. Provided that the r-dependence of these fields is controlled by the RG flow equations, we have found the exact solution to the 11-dimensional field equations. With this solution, one would say that the $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$-invariant holographic RG flow can be lifted to an $\mathcal{N} = 2$ M2-brane flow in M-theory. The field strengths must be subject to the nontrivial boundary conditions at both UV and IR critical points.

It is natural to ask what is corresponding dual gauge theory for the previous 11-dimensional background in the context of AdS/CFT. According to the observation of $T^{1,1}$-flow [14], the quiver $\mathrm{U}(N)^3$ Chern-Simons gauge theory for the M2-branes probing the cone over $Q^{1,1,1}$ space provides the quiver diagram for a partial resolution [23] of $Q^{1,1,1}$ theory with $\mathrm{U}(N)^3$ gauge group and two SU(2) doublets and an adjoint field. It is known that in [24], the higher dimensional analog of the $Y^{p,q}$ space was found(and denoted by $X^{p,q}$) and can be expressed as a U(1) bundle over 6-dimensional Einstein-Kahler space which is a 2-bundle over a 4-dimensional Einstein-Kahler space. Therefore, the partial resolution of the $X^{p,q}$ might be a candidate for the dual gauge theory. The spin-2 KK modes around a warped product of AdS_4 and a squashed and stretched 7-manifold can be obtained. The

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mass-squared in AdS_4, in principle, can be determined and it is an open problem to find out what $\mathcal{N}=2$ SCFT operators in Chern-Simons matter theory are.

As mentioned in [14], one can study the other possibility where there exists a bigger $\mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$ symmetry for the 11-dimensional lift of the same RG flow equations we discussed in this paper. For the $\mathbf{CP}^2$ choice for the Einstein-Kahler 2-fold inside of $X^{p,q}$ space, the isometry is given by $\mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)_R$. In the sense that this has two U(1)'s, the construction for the 4-form field strengths is similar to each other. That is, among three U(1) symmetries, only two U(1)'s are preserved. Then it is nontrivial to find out the 4-forms which should preserve these symmetries explicitly.

There exists $\mathcal{N}=1$ G_2 critical point in 4-dimensional gauged supergravity. That is, this IR critical point is located at some point in the lower plane of figure 1. The 11-dimensional lift of this theory, which is present in the upper plane of figure 1, was found in [6], as mentioned in the introduction. One can think of other 11-dimensional solution with same RG flow equations for the AdS_4 supergravity fields. Inside of 7-dimensional ellipsoid, there exists a round six-sphere $\mathbf{S}^6$ which has SO(7) symmetry. It is an open problem whether one can embed the appropriate Einstein-Kahler 2-fold inside of $\mathbf{S}^6$. Of course, the original global symmetry G_2 should break into a smaller group symmetry.

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A The Ricci tensor in frame basis

The 11-dimensional metric (2.1) with (2.10) and (2.13) generates the Ricci tensor in frame basis as follows:

$$
\begin{aligned}
R_1^1 &= -\frac{1}{9\sqrt{3}\hat{L}^2 u^{\frac{2}{3}} v^{\frac{4}{3}} (c_\mu^2+u^2 s_\mu^2)^{\frac{8}{3}}} 2\left[2u^8 v^2 (v^2-1) s_\mu^4 + 2v^2(v^2+3)c_\mu^4\right. \\
&\quad + u^6\left[-2(-5+c_{2\mu}) + v^2(-11+c_{2\mu}) + 4v^4 c_\mu^2\right] s_\mu^2 \\
&\quad \left. + u^2\left[12c_\mu^2 + v^2(9-13c_{2\mu}) + 4v^4 s_\mu^2\right] c_\mu^2 + u^4\left[6s_{2\mu}^2 + v^2(5-8c_{2\mu}+5c_{4\mu}) + v^4 s_{2\mu}^2\right]\right] \\
&= R_2^2 = R_3^3 = -2R_6^6 = -2R_7^7 = -2R_8^8 = -2R_9^9, \\
R_4^4 &= \frac{1}{18\sqrt{3}\hat{L}^2 u^{\frac{2}{3}} v^{\frac{10}{3}} (c_\mu^2+u^2 s_\mu^2)^{\frac{8}{3}}}\left[-4v^2(2v^4-21v^2+27)c_\mu^4 - 4u^8 v^2(2v^4-5v^2+3)s_\mu^4\right. \\
&\quad + 2u^2 v^2\left[-48+60c_{2\mu} + v^2(15-43c_{2\mu}) + 4v^4 s_\mu^2\right] c_\mu^2 \\
&\quad - 2u^6\left[24c_\mu^2 - 4v^2(7+4c_{2\mu}) + v^4(17+5c_{2\mu}) - 4v^6 c_\mu^2\right] s_\mu^2 \\
&\quad \left. - u^4 v^2\left(33-48c_{2\mu}+27c_{4\mu}+4v^2\left[2+4c_{2\mu}-7c_{4\mu}+v^2(-1+c_{4\mu})\right]\right)\right], \\
R_4^5 &= \frac{1}{6\sqrt{3}\hat{L}^2 v^{\frac{1}{3}} (c_\mu^2+u^2 s_\mu^2)^{\frac{8}{3}}} u^{\frac{1}{3}}\left(-2c_\mu^2(v^2-3)\right. \\
&\quad \left. + u^2\left[-5-11c_{2\mu}+14v^2 c_\mu^2 + u^2\left(-11+9c_{2\mu}+2s_\mu^2\left[u^2-v^2(u^2-7)\right]\right)\right]\right) s_{2\mu} = R_5^4,
\end{aligned}
$$

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$$
\begin{aligned}
R^5_5 &= \frac{1}{18\sqrt{3}\hat{L}^2 u^{\frac{2}{3}} v^{\frac{4}{3}} (c_\mu^2 + u^2 s_\mu^2)^{\frac{8}{3}}} \left[4u^8 v^2 (v^2-1) s_\mu^4 + 4v^2(v^2+3)c_\mu^4\right. \\
&\quad + u^4\left[6 - 6c_{4\mu} + v^2(19 - 16c_{2\mu} + c_{4\mu}) + 5v^4(-1+c_{4\mu})\right] \\
&\quad \left. + \, 4u^2\left[6c_\mu^2 + v^2(3 - !5c_{2\mu}) + 2v^4 s_\mu^2\right] c_\mu^2 + 4u^6\left[-1 - v^2 + v^4 + (-7+5v^2+v^4)c_{2\mu}\right] s_\mu^2\right], \\
R^{10}_{10} &= \frac{1}{18\sqrt{3}\hat{L}^2 u^{\frac{2}{3}} v^{\frac{10}{3}} (c_\mu^2 + u^2 s_\mu^2)^{\frac{8}{3}}} \left[4u^8 v^4 (v^2-1) s_\mu^4 + 4v^4(v^2+3)c_\mu^4\right. \\
&\quad + u^4 v^4\left[-2(-11+8c_{2\mu}+c_{4\mu}) + 5v^2(-1+c_{4\mu})\right] + 2u^2 v^2 c_\mu^2\left[12c_\mu^2 + v^2(9-13c_{2\mu}) + 4v^4 s_\mu^2\right] \\
&\quad \left. + \, 2u^6\left[24c_\mu^2 - 2v^2(7+13c_{2\mu}) + v^4(1+13c_{2\mu}) + 4v^6 c_\mu^2\right] s_\mu^2\right], \\
R^{11}_{10} &= -\frac{2u^{\frac{7}{3}}(v^2-1)(u^2+2v^2-3)s_{2\mu}}{3\sqrt{3}\hat{L}^2 v^{\frac{7}{3}} (c_\mu^2 + u^2 s_\mu^2)^{\frac{5}{3}}} = R^{10}_{11}, \\
R^{11}_{11} &= \frac{1}{9\sqrt{3}\hat{L}^2 u^{\frac{2}{3}} v^{\frac{10}{3}} (c_\mu^2 + u^2 s_\mu^2)^{\frac{8}{3}}} \left[2u^8 v^4 (v^2-1) s_\mu^4 + 2v^4(v^2+3)c_\mu^4\right. \\
&\quad + u^2 v^2\left[-24c_\mu^2 + v^2(27+5c_{2\mu}) + 4v^4 s_\mu^2\right] c_\mu^2 + u^4 v^2\left[-6s_{2\mu}^2 + 2v^2(4-4c_{2\mu}+c_{4\mu}) + 7v^4 s_{2\mu}^2\right] \\
&\quad \left. + \, u^6\left[-24c_\mu^2 + 8v^2(2+5c_{2\mu}) - v^4(5+29c_{2\mu}) + 4v^6 c_\mu^2\right] s_\mu^2\right],
\end{aligned}
\tag{A.1}
$$

where we introduce

$$
u(r) \equiv \rho(r)^4, \qquad v(r) \equiv \cosh\chi(r).
$$

We also use a simplified notation for the trigonometric function as in $s_\mu \equiv \sin\mu$ and so on. By substituting the IR fixed point values (2.17) into (A.1), one sees R^5_4 vanishes and the other components reduce to (2.20).

B The 4-form field strength in frame basis

One can read off the 4-forms from (2.19), (2.22) and (2.26) and they are given in the frame basis as follows:

$$
\begin{aligned}
F_{1234} &= \frac{3^{\frac{1}{4}}\left[c_\mu^2(-5+\cosh 2\chi) + 2\rho^8(-2+c_{2\mu}+s_\mu^2\,\rho^8\,\sinh^2\chi)\right]}{\hat{L}\,\rho^{\frac{4}{3}}\,\cosh^{\frac{2}{3}}\chi\,(c_\mu^2+\rho^8\,s_\mu^2)^{\frac{4}{3}}}, \\
F_{1235} &= \frac{3^{\frac{1}{4}}\,\rho^{\frac{8}{3}}\,\left[1+\cosh 2\chi + \rho^8\,(-3+\cosh 2\chi)\right]}{\hat{L}\,\cosh^{\frac{5}{3}}\chi\,(c_\mu^2+\rho^8\,s_\mu^2)^{\frac{4}{3}}}\, s_\mu\, c_\mu, \\
F_{4568} &= -\frac{3^{\frac{1}{4}}\,(-3+\rho^8)\,\sinh 2\chi}{2\hat{L}\,\rho^{\frac{4}{3}}\,\cosh^{\frac{5}{3}}\chi\,(c_\mu^2+\rho^8\,s_\mu^2)^{\frac{1}{3}}}\, c_{\alpha+\psi} = -F_{4579} = F_{469\,10} = F_{478\,10}, \\
F_{4569} &= \frac{3^{\frac{1}{4}}\,(-3+\rho^8)\,\sinh 2\chi}{2\hat{L}\,\rho^{\frac{4}{3}}\,\cosh^{\frac{5}{3}}\chi\,(c_\mu^2+\rho^8\,s_\mu^2)^{\frac{1}{3}}}\, s_{\alpha+\psi} = -F_{468\,10} = F_{4578} = F_{479\,10}, \\
F_{468\,11} &= -\frac{3^{\frac{1}{4}}\,\rho^{\frac{8}{3}}\,\mathrm{sech}^{\frac{5}{3}}\chi\,\sinh\chi\,\left[1+\cosh 2\chi+\rho^8\,(-3+\cosh 2\chi)\right]}{2\hat{L}\,(c_\mu^2+\rho^8\,s_\mu^2)^{\frac{4}{3}}}\, s_{2\mu}\, s_{\alpha+\psi} \\
&= -F_{479\,11},
\end{aligned}
$$

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$$
\begin{aligned}
F_{469\,11} &= -\frac{3^{\frac{1}{4}}\,\rho^{\frac{8}{3}}\,\mathrm{sech}^{\frac{5}{3}}\chi\;\sinh\chi\;\left[1+\cosh 2\chi+\rho^{8}\,(-3+\cosh 2\chi)\right]}{2\hat{L}\,(c_{\mu}^{2}+\rho^{8}\,s_{\mu}^{2})^{\frac{4}{3}}}\,s_{2\mu}\,c_{\alpha+\psi} \\
&= F_{478\,11}, \\
F_{568\,10} &= \frac{3^{\frac{1}{4}}\,\rho^{\frac{8}{3}}\,(-1+\rho^{8})\,\sinh\chi}{\hat{L}\,\mathrm{sech}^{\frac{1}{3}}\chi\,(c_{\mu}^{2}+\rho^{8}\,s_{\mu}^{2})^{\frac{4}{3}}}\,s_{2\mu}\,s_{\alpha+\psi} = -F_{579\,10}, \\
F_{568\,11} &= -\frac{3^{\frac{1}{4}}\,(3+\rho^{8})\,\sinh\chi}{\hat{L}\,\rho^{\frac{4}{3}}\,\cosh^{\frac{2}{3}}\chi\,(c_{\mu}^{2}+\rho^{8}\,s_{\mu}^{2})^{\frac{1}{3}}}\,s_{\alpha+\psi} = -F_{579\,11} = F_{69\,10\,11} = F_{78\,10\,11}, \\
F_{569\,10} &= \frac{3^{\frac{1}{4}}\,\rho^{\frac{8}{3}}\,(-1+\rho^{8})\,\sinh\chi}{\hat{L}\,\mathrm{sech}^{\frac{1}{3}}\chi\,(c_{\mu}^{2}+\rho^{8}\,s_{\mu}^{2})^{\frac{4}{3}}}\,c_{\alpha+\psi}\,s_{2\mu} = F_{578\,10}, \\
F_{569\,11} &= -\frac{3^{\frac{1}{4}}\,(3+\rho^{8})\,\sinh\chi}{\hat{L}\,\rho^{\frac{4}{3}}\,\cosh^{\frac{2}{3}}\chi\,(c_{\mu}^{2}+\rho^{8}\,s_{\mu}^{2})^{\frac{1}{3}}}\,c_{\alpha+\psi} = F_{578\,11} = -F_{68\,10\,11} = F_{79\,10\,11}. \qquad \text{(B.1)}
\end{aligned}
$$

For simplicity, we ignored the r dependence on ρ and χ in the right hand side of (B.1). When we substitute the UV fixed point value (2.15) into (B.1), then only F_{1234} is nonzero. When we substitute the IR fixed point values (2.17) into (B.1), one sees $F_{\mu\nu\rho 5}, F_{4mnp}$ and F_{45mn} where $\mu,\nu,\rho = 1,2,3$ and $m,n,p = 6,7,\cdots,11$ vanish due to the $\sinh\chi(r)$ and the other components reduce to (2.21). These vanishing 4-forms have either $1+\cosh 2\chi + \rho^{8}\,(-3+\cosh 2\chi)$, which leads to $(3-\rho^{8})$ for the condition $\cosh 2\chi = 2$, or $(-3+\rho^{8})$. This nontrivial boundary conditions also occur for the $\mathcal{N}=1$ G_2 M2-brane flow in 11-dimensions [6].

References

[1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

[2] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080] [SPIRES].

[3] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [SPIRES];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [SPIRES].

[4] C.-h. Ahn and J. Paeng, *Three-dimensional SCFTs, supersymmetric domain wall and renormalization group flow*, *Nucl. Phys.* **B 595** (2001) 119 [hep-th/0008065] [SPIRES].

[5] C. Ahn and K. Woo, *Supersymmetric domain wall and RG flow from 4-dimensional gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 599** (2001) 83 [hep-th/0011121] [SPIRES];
C. Ahn and S.-J. Rey, *More CFTs and RG flows from deforming M2/M5-brane horizon*, *Nucl. Phys.* **B 572** (2000) 188 [hep-th/9911199] [SPIRES].

JHEP11(2009)022

[6] C.-h. Ahn and T. Itoh, *An $N = 1$ supersymmetric G_2-invariant flow in M-theory*, Nucl. Phys. **B 627** (2002) 45 [hep-th/0112010] [SPIRES].

[7] R. Corrado, K. Pilch and N.P. Warner, *An $N = 2$ supersymmetric membrane flow*, Nucl. Phys. **B 629** (2002) 74 [hep-th/0107220] [SPIRES].

[8] C. Ahn, *Holographic Supergravity Dual to Three Dimensional $N = 2$ Gauge Theory*, JHEP **08** (2008) 083 [arXiv:0806.1420] [SPIRES].

[9] M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons Theories and AdS_4/CFT_3 Correspondence*, JHEP **09** (2008) 072 [arXiv:0806.1519] [SPIRES];
I. Klebanov, T. Klose and A. Murugan, *AdS_4/CFT_3 – Squashed, Stretched and Warped*, JHEP **03** (2009) 140 [arXiv:0809.3773] [SPIRES];
I.R. Klebanov, S.S. Pufu and F.D. Rocha, *The Squashed, Stretched and Warped Gets Perturbed*, JHEP **06** (2009) 019 [arXiv:0904.1009] [SPIRES].

[10] C. Ahn, *Towards Holographic Gravity Dual of N=1Superconformal Chern-Simons Gauge Theory*, JHEP **07** (2008) 101 [arXiv:0806.4807] [SPIRES]; *Comments on Holographic Gravity Dual of $N = 6$ Superconformal Chern-Simons Gauge Theory*, JHEP **03** (2009) 107 [arXiv:0812.4363] [SPIRES].

[11] N. Bobev, N. Halmagyi, K. Pilch and N.P. Warner, *Holographic, $N = 1$ Supersymmetric RG Flows on M2 Branes*, JHEP **09** (2009) 043 [arXiv:0901.2736] [SPIRES].

[12] C. Ahn and K. Woo, *N=8 Gauged Supergravity Theory and $N = 6$ Superconformal Chern-Simons Matter Theory*, arXiv:0901.4666 [SPIRES];
C. Ahn and K. Woo, *Are There Any New Vacua of Gauged $N = 8$ Supergravity in Four Dimensions?*, arXiv:0904.2105 [SPIRES];
C. Ahn, *The Gauge Dual of Gauged $N = 8$ Supergravity Theory*, arXiv:0905.3943 [SPIRES].

[13] C. Ahn and K. Woo, *Perturbing Around A Warped Product Of AdS_4 and Seven-Ellipsoid*, JHEP **08** (2009) 065 [arXiv:0907.0969] [SPIRES].

[14] C. Ahn and K. Woo, *The Gauge Dual of A Warped Product of AdS_4 and A Squashed and Stretched Seven-Manifold*, arXiv:0908.2546 [SPIRES].

[15] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, Class. Quant. Grav. **1** (1984) 499 [SPIRES];
C. Ahn, *Squashing Gravity Dual of $N = 6$ Superconformal Chern-Simons Gauge Theory*, Class. Quant. Grav. **26** (2009) 105001 [arXiv:0809.3684] [SPIRES].

[16] C.N. Pope and N.P. Warner, *An SU(4) invariant compactification of $D = 11$ supergravity on a stretched seven sphere*, Phys. Lett. **B 150** (1985) 352 [SPIRES];
C. Ahn and T. Itoh, *The 11-dimensional metric for AdS/CFT RG flows with common SU(3) invariance*, Nucl. Phys. **B 646** (2002) 257 [hep-th/0208137] [SPIRES];
C. Ahn, *Other Squashing Deformation and $N = 3$ Superconformal Chern-Simons Gauge Theory*, Phys. Lett. **B 671** (2009) 303 [arXiv:0810.2422] [SPIRES].

[17] M. Cvetič, H. Lü and C.N. Pope, *Consistent warped-space Kaluza-Klein reductions, half-maximal gauged supergravities and CP(n) constructions*, Nucl. Phys. **B 597** (2001) 172 [hep-th/0007109] [SPIRES].

[18] P. Candelas and X.C. de la Ossa, *Comments on Conifolds*, Nucl. Phys. **B 342** (1990) 246 [SPIRES].

JHEP11(2009)022

[19] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Sasaki-Einstein metrics on* $S^2 \times S^3$, *Adv. Theor. Math. Phys.* **8** (2004) 711 [hep-th/0403002] [SPIRES].

[20] B. de Wit, H. Nicolai and N.P. Warner, *The embedding of gauged* $N = 8$ *supergravity into* $D = 11$ *supergravity*, *Nucl. Phys.* **B 255** (1985) 29 [SPIRES].

[21] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409 [SPIRES].

[22] P.G.O. Freund and M.A. Rubin, *Dynamics of Dimensional Reduction*, *Phys. Lett.* **B 97** (1980) 233 [SPIRES].

[23] S. Franco, I.R. Klebanov and D. Rodriguez-Gomez, *M2-branes on Orbifolds of the Cone over* $Q^{1,1,1}$, *JHEP* **08** (2009) 033 [arXiv:0903.3231] [SPIRES].

[24] J.P. Gauntlett, D. Martelli, J.F. Sparks and D. Waldram, *A new infinite class of Sasaki-Einstein manifolds*, *Adv. Theor. Math. Phys.* **8** (2006) 987 [hep-th/0403038] [SPIRES].

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